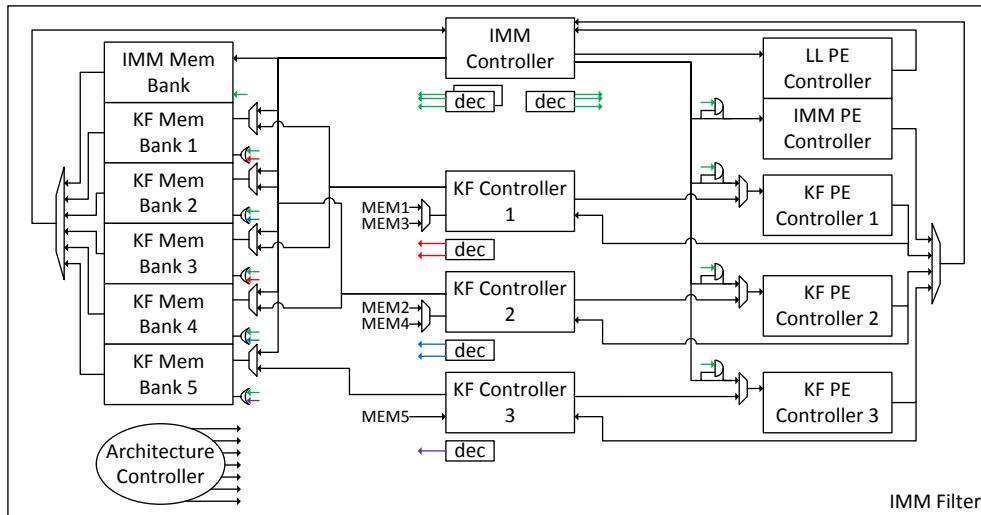


# Implementation of a Parameterized Interacting Multiple Model Filter on an FPGA for Satellite Communications



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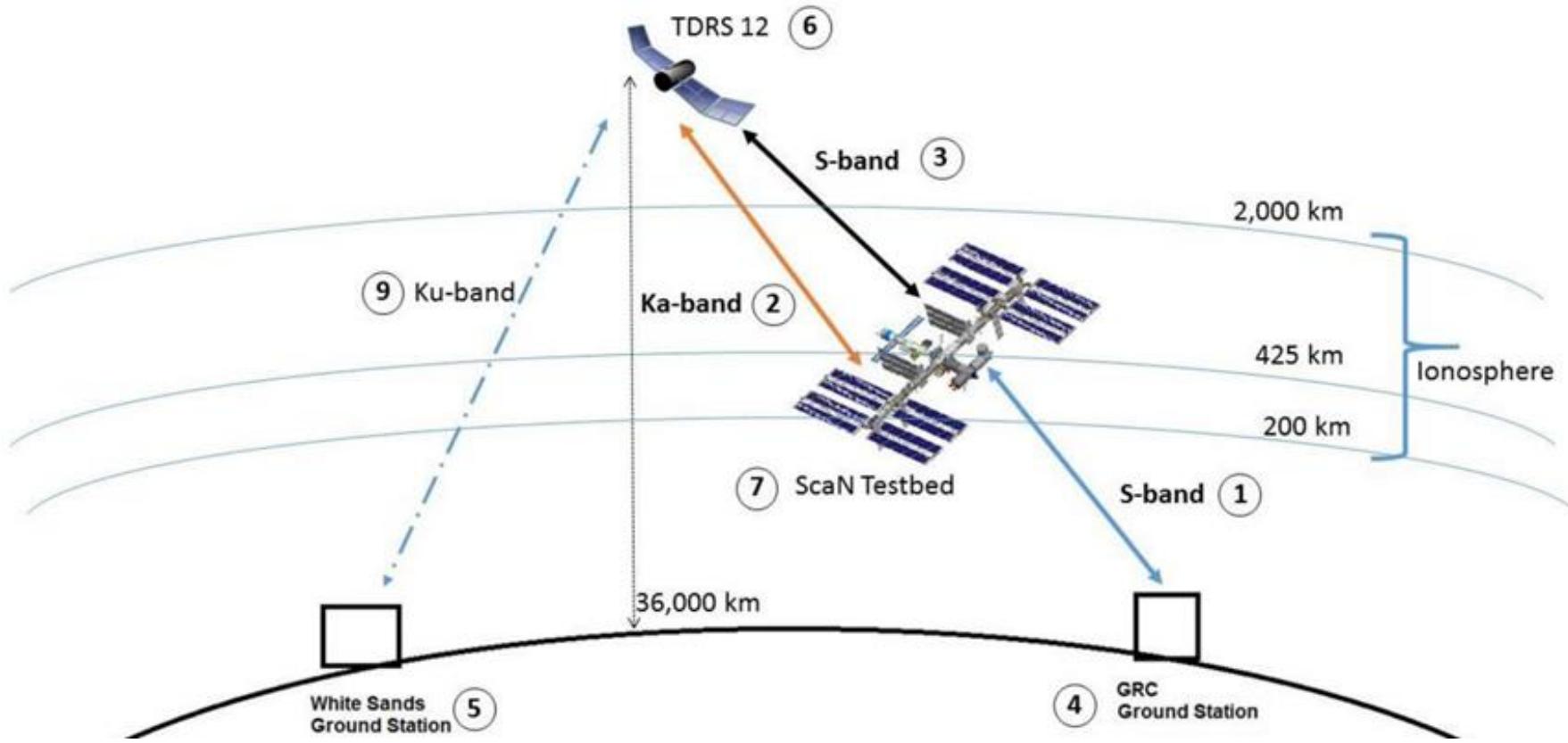
16 November 2016



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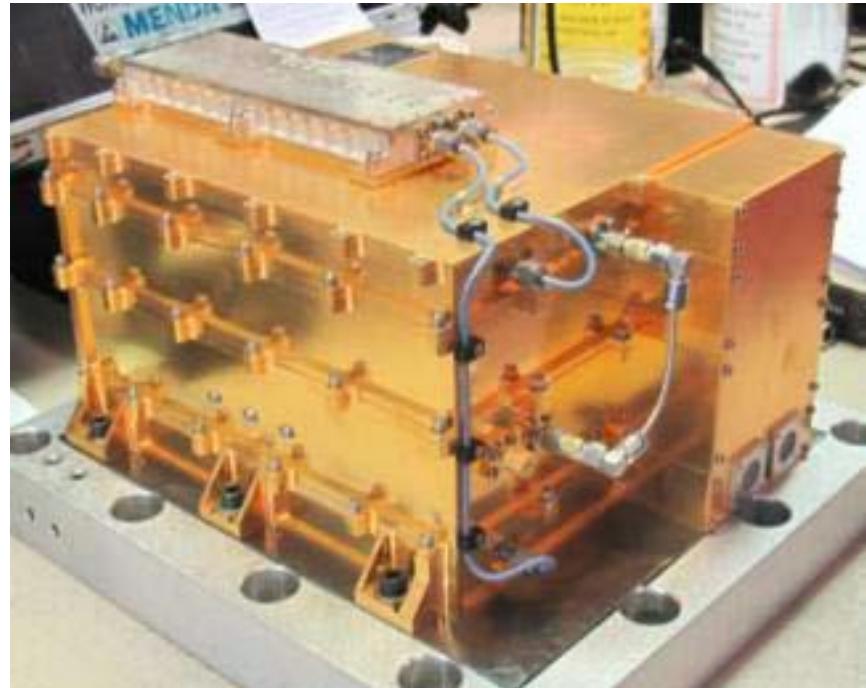
# Overview of Research



# Targeted Platform

## JPL Radio

- **66-MHz SPARC processor**
- **2 x Xilinx Virtex-II FPGAs**
- **S-band transceiver**
- **L-Band (GPS) receiver**

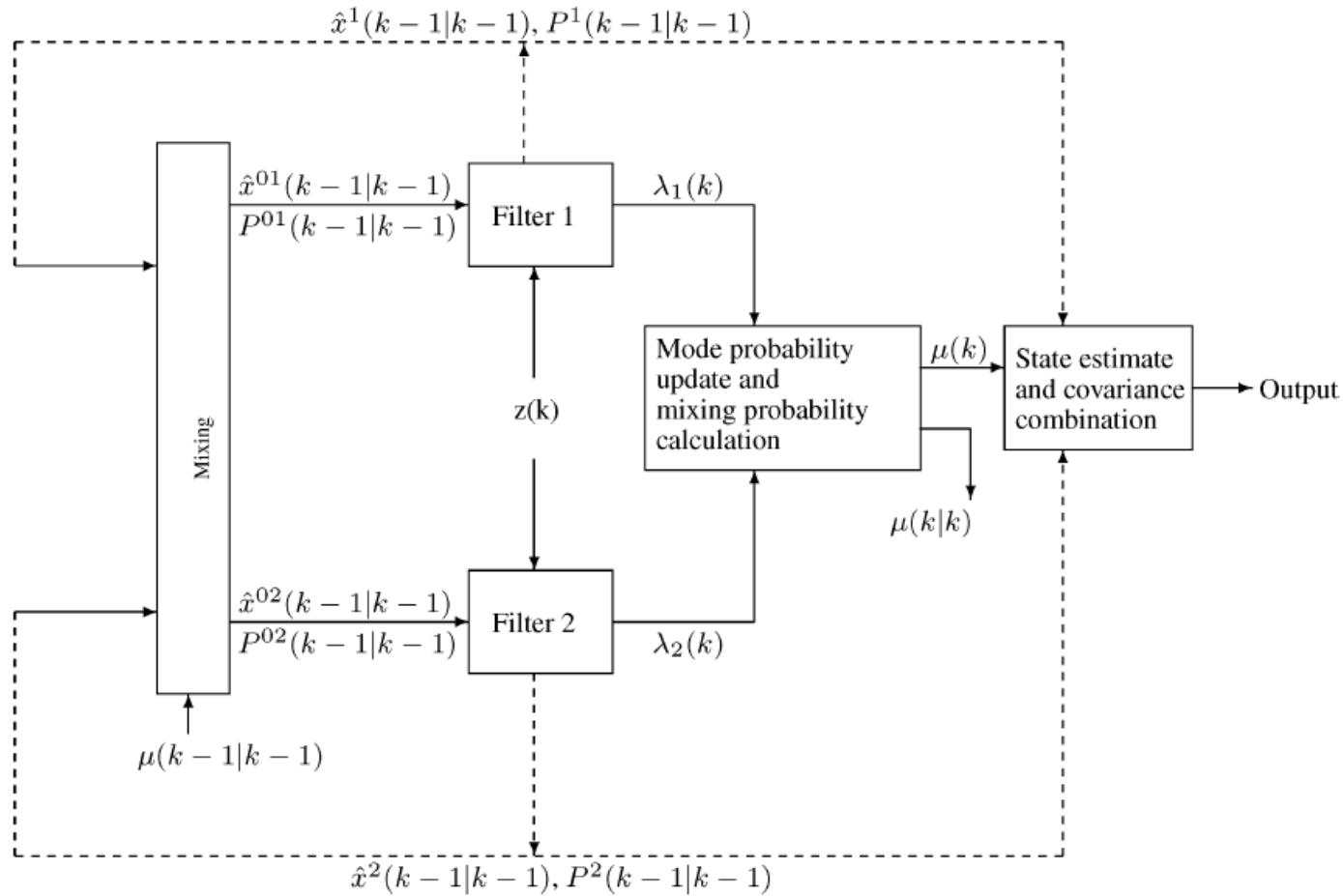


Picture Source: Reinhart, R., *Wireless Innovation Forum Technical Conference*, 2013



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# Interacting Multiple Model (IMM) Background



Picture Source: Sathyan, T. and Kirubarajan, T., *IEEE Transactions on Circuits and Systems, 2006*



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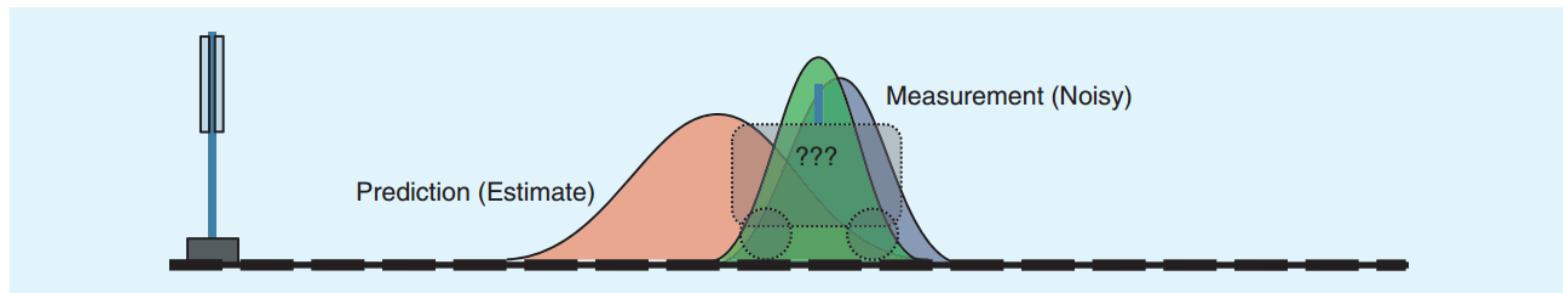
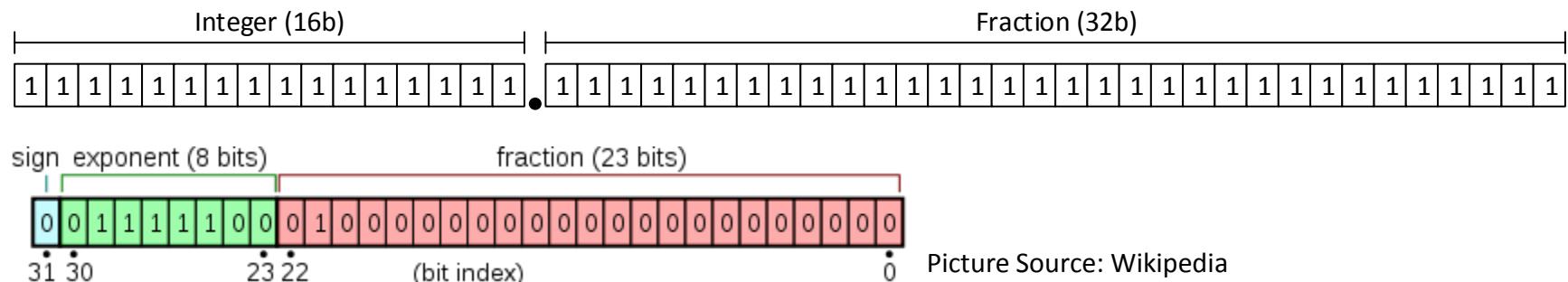
# Interacting Multiple Model (IMM) Background

Step Number	Step Type	Iterations Needed	Expression Executed
MIX01	SF	1	$\hat{\mu}(k+1 k) = \underline{\mu}^T(k) \times \pi^T$
MIX02	SF	1	$\pi^T \times \text{diag}(\underline{\mu}(k))$
MIX03	SF	1	$\underline{\mu}(k) = (\pi^T \times \text{diag}(\underline{\mu}(k)))^T \times (\text{diag}(\hat{\mu}(k+1 k)))^{-1}$
MIX04	SF	1	$\hat{X}_0(k k) = \hat{X}(k k) \times \underline{\mu}(k)$
MIX05	DF	$R^2$	$\hat{X}_{j0}(k k) - \hat{X}_i(k k)$
MIX06	DF	$R^2$	$(\hat{X}_{j0}(k k) - \hat{X}_i(k k)) \times (\hat{X}_{j0}(k k) - \hat{X}_i(k k))^T + P_i(k k)$
MIX07	DF	$R^2$	$P_{j0}(k k) = [(\hat{X}_{j0}(k k) - \hat{X}_i(k k)) \times (\hat{X}_{j0}(k k) - \hat{X}_i(k k))^T + P_i] \times \text{diag}(\underline{\mu}_{ij}(k)) + G$
KF01	DF	$R$	$\hat{X}_j(k+1 k) = F \times \hat{X}_{j0}(k k)$
KF02	DF	$R$	$F \times P_{j0}(k k)$
KF03	DF	$R$	$P_j(k+1 k) = F \times P_{j0}(k k) \times F^T + Q$
KF04	DF	$R$	$P_j(k+1 k) \times H_j^T$
KF05	DF	$R$	$S_j(k+1) = H \times P_j(k+1 k) \times H^T + R_j$
KF06	DF	$R$	$K_j(k+1) = P_j(k+1 k) \times H^T \times (S_j(k+1))^{-1}$
KF07	DF	$R$	$P_j(k+1 k+1) = P_j(k+1 k) - K_j(k+1) \times (P_j(k+1 k) \times H_j^T)^T$
KF08	DF	$R$	$e_j(k+1) = \underline{z}(k+1) - H \times \hat{X}_j(k+1 k)$
KF09	DF	$R$	$\hat{X}_j(k+1 k+1) = \hat{X}_j(k+1 k) + K_j(k+1) \times \underline{e}_j(k+1)$
COM01	DF	$R$	$\underline{e}_j^T(k+1) \times S_j^{-1}(k+1) \times \underline{e}_j(k+1)$
COM02	SC	$R$	$-\frac{1}{2} \underline{e}_j^T(k+1) \times S_j^{-1}(k+1) \times e_j(k+1)$
COM03	SC	$R$	$\exp(-\frac{1}{2} \underline{e}_j^T(k+1) \times S_j^{-1}(k+1) \times e_j(k+1))$
COM04	SC	$R$	$ S_j(k+1) $
COM05	SC	$R$	$( S_j(k+1) )^{-\frac{1}{2}}$
COM06	SC	$R$	$\underline{L}_j(k+1) = (2\pi)^{\frac{d}{2}} ( S_j(k+1) )^{-\frac{1}{2}} \exp(-\frac{1}{2} \underline{e}_j^T(k+1) \times S_j^{-1}(k+1) \times e_j(k+1))$
COM07	SF	1	$\text{diag}(\underline{L}(k+1)) \times \hat{\mu}^T(k+1 k)$
COM08	SF	1	$\hat{\mu}(k+1 k) \times \underline{L}^T(k+1)$
COM09	SF	1	$\underline{\mu}(k+1) = (\text{diag}(\hat{\mu}(k+1 k) \times \underline{L}^T(k+1)))^{-1} \times (\text{diag}(\underline{L}(k+1)) \times \hat{\mu}^T(k+1 k))$
COM10	SF	1	$\hat{X}(k+1 k+1) = \hat{X}(k+1 k+1) \times \underline{\mu}(k+1)$



# Implementation Trade Studies

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \quad A^{-1} = ?$$

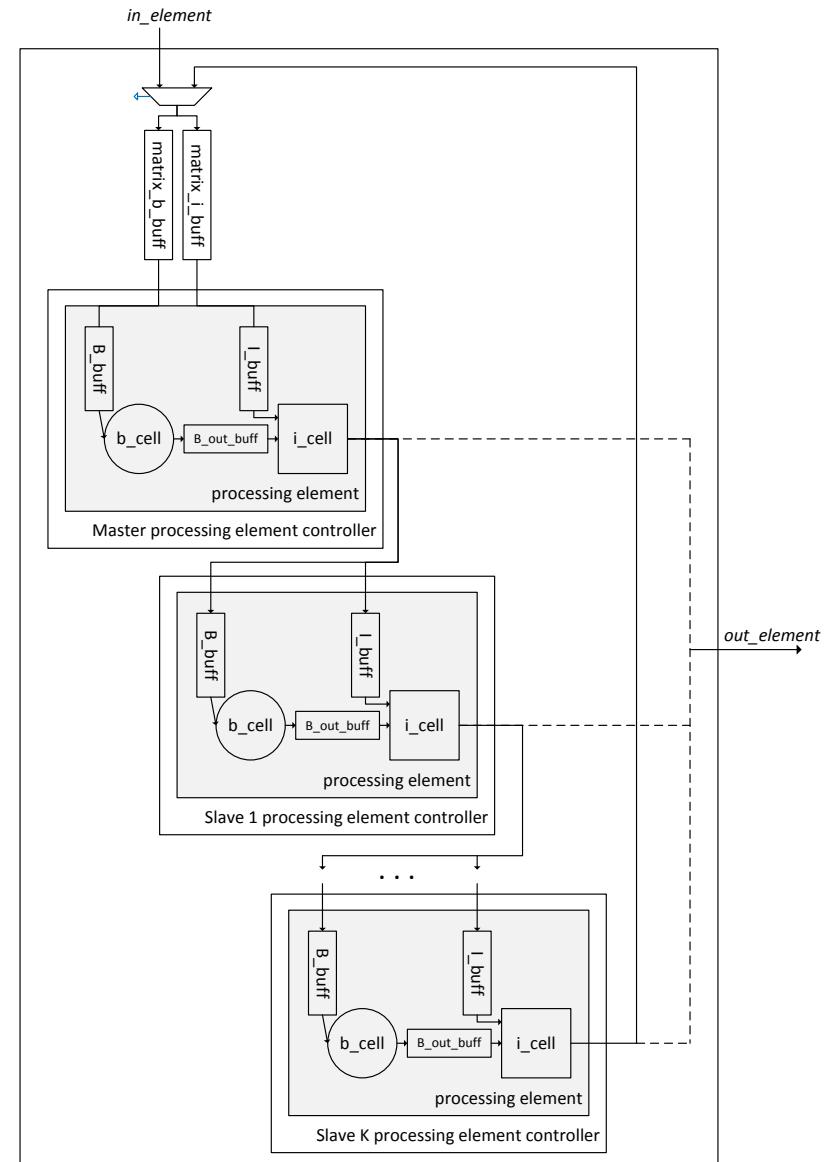
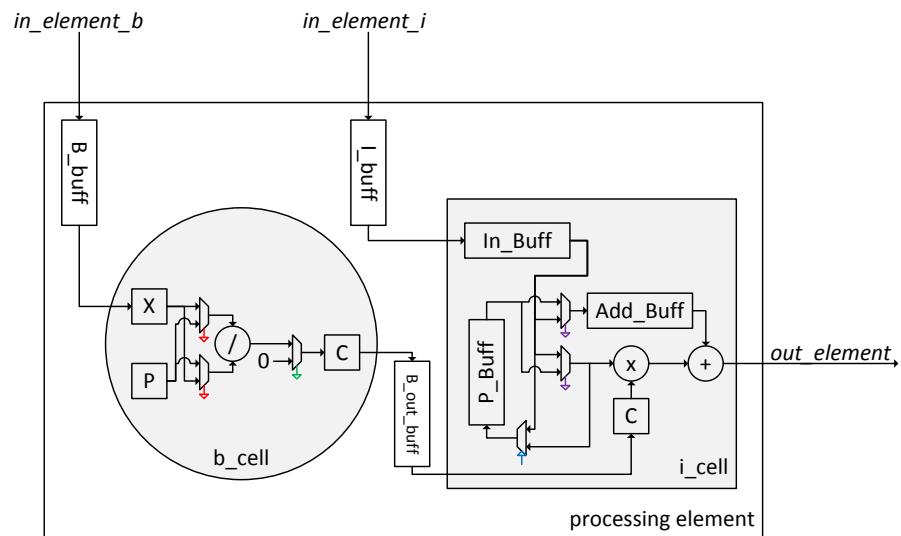


Picture Source: Faragher, R., *IEEE Signal Processing Magazine*, 2012



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# Schur Complement Architecture

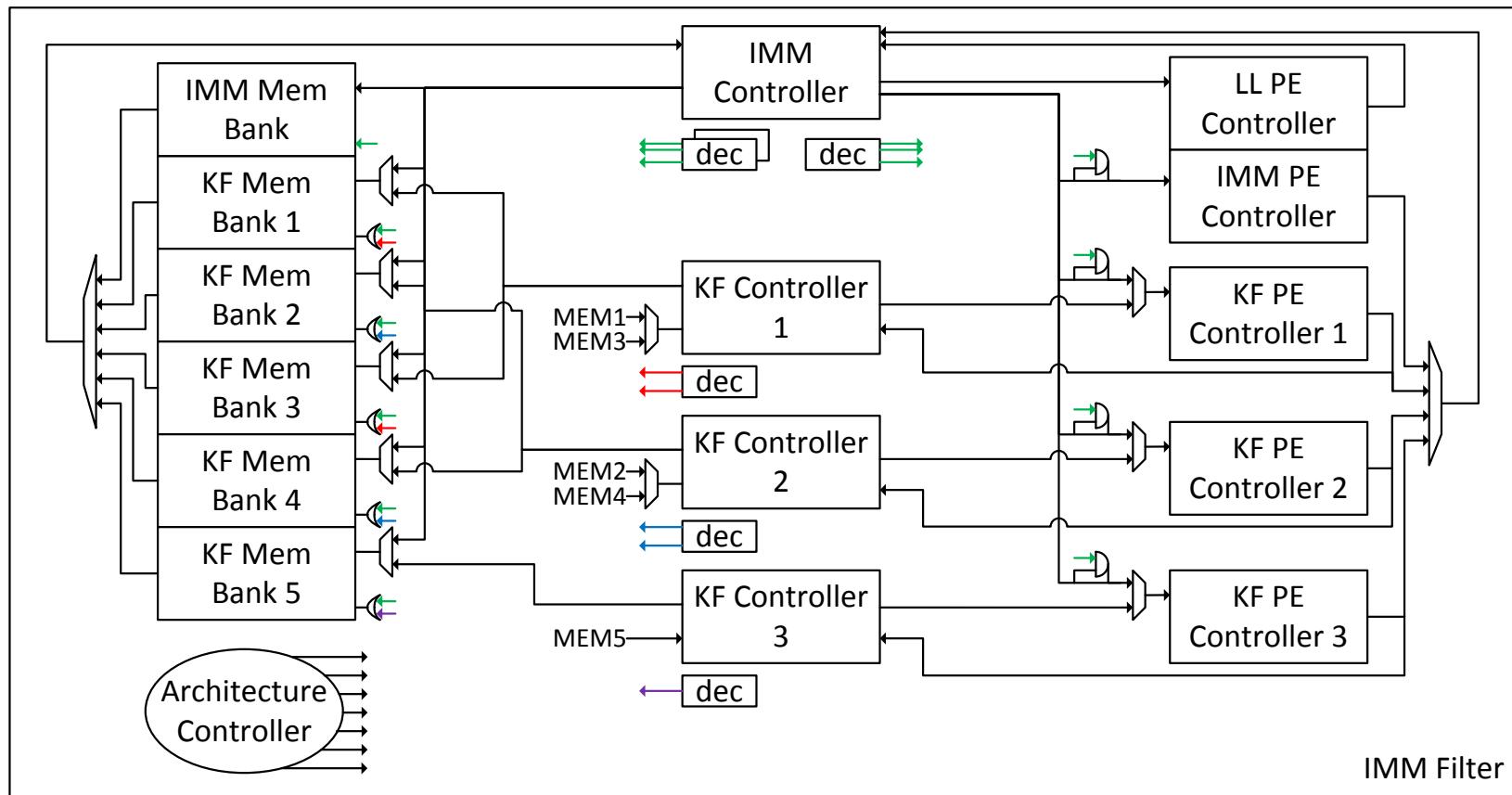


# Schur Complement Analysis

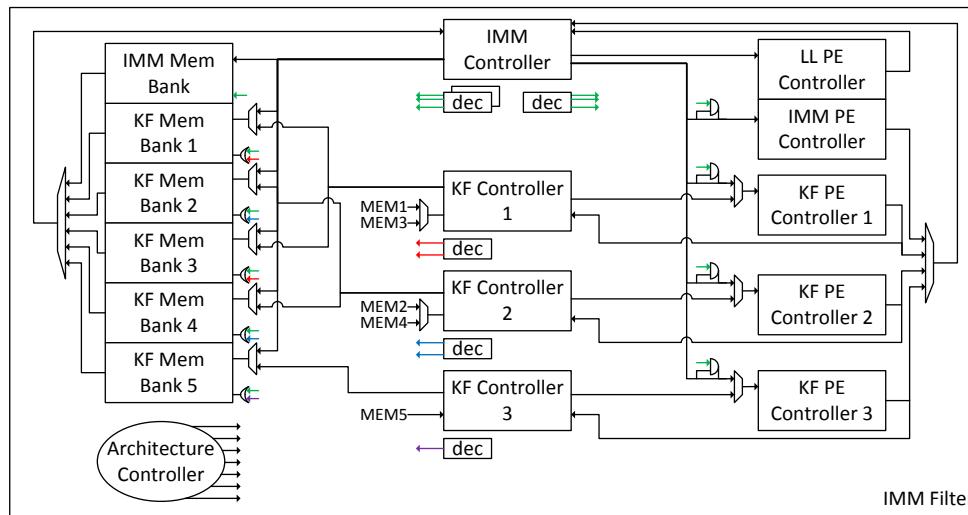
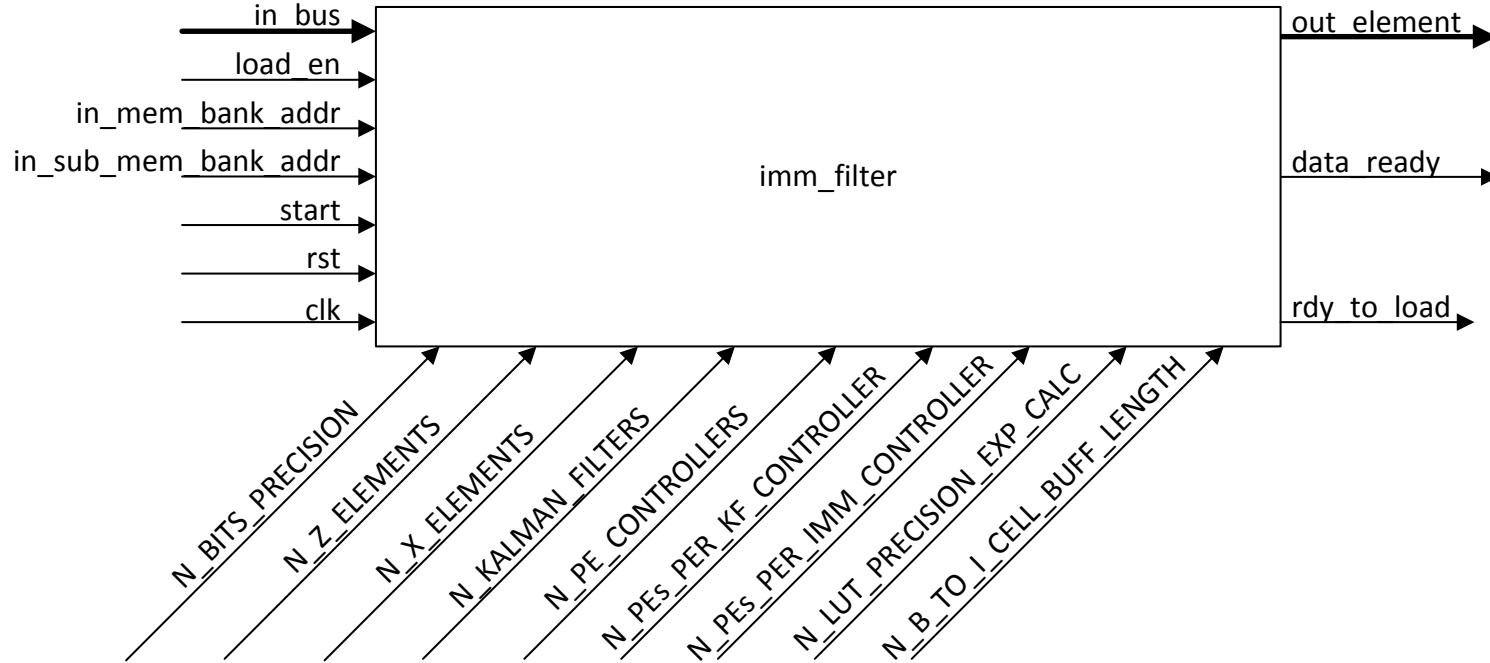
Multiplier Latency	Addition Latency	Divider Cycles Per Operation	Divider Latency	Occupied Slices	Slice Flip Flops	4-Input LUTs	Max Clock Frequency (MHz)
1	1	14	14	754	541	1299	50.742
3	5	14	14	829	925	1340	133.233
3	7	26	27	849	1090	1328	152.879
6	7	26	27	910	1201	1343	175.389



# IMM Filter Architecture



# IMM Analysis



# Example Scenario

## State Model

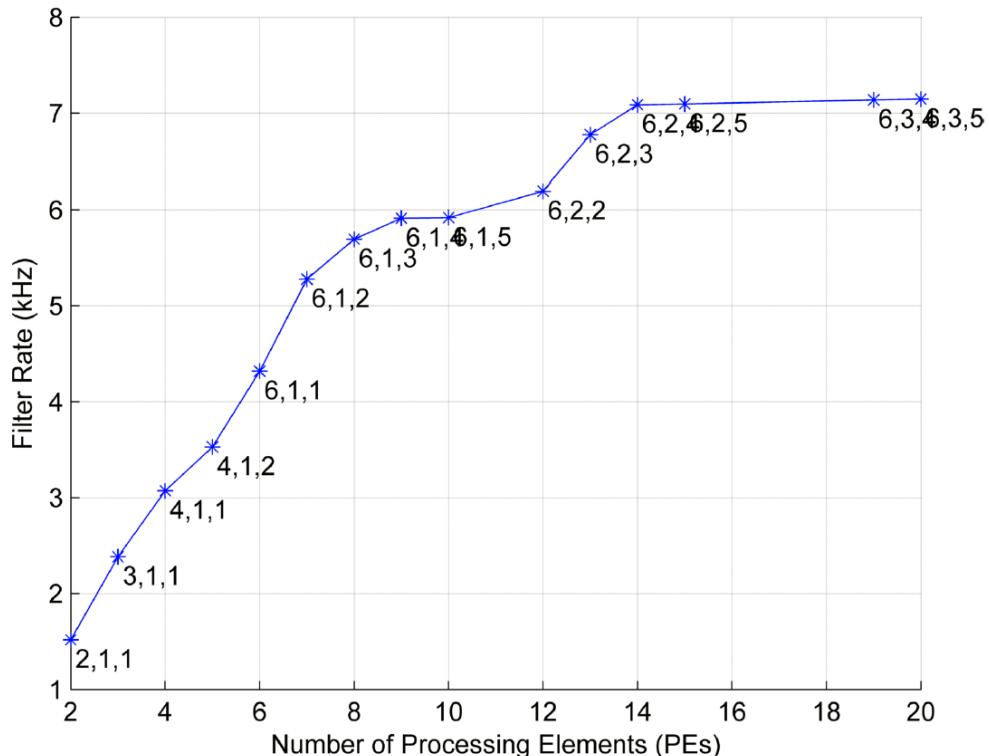
## Constant Acceleration Model

## Available Measurements

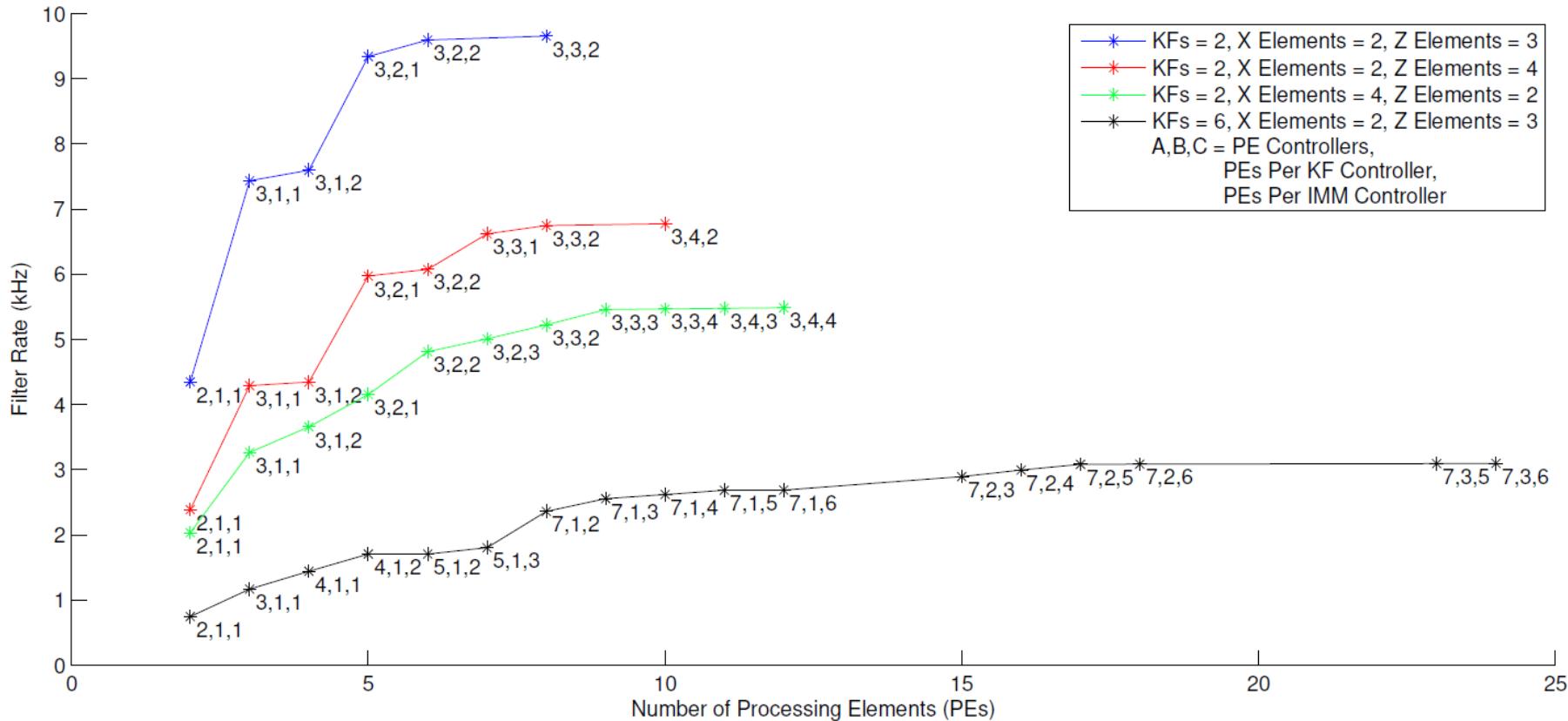
SNR, SIR

## System Models

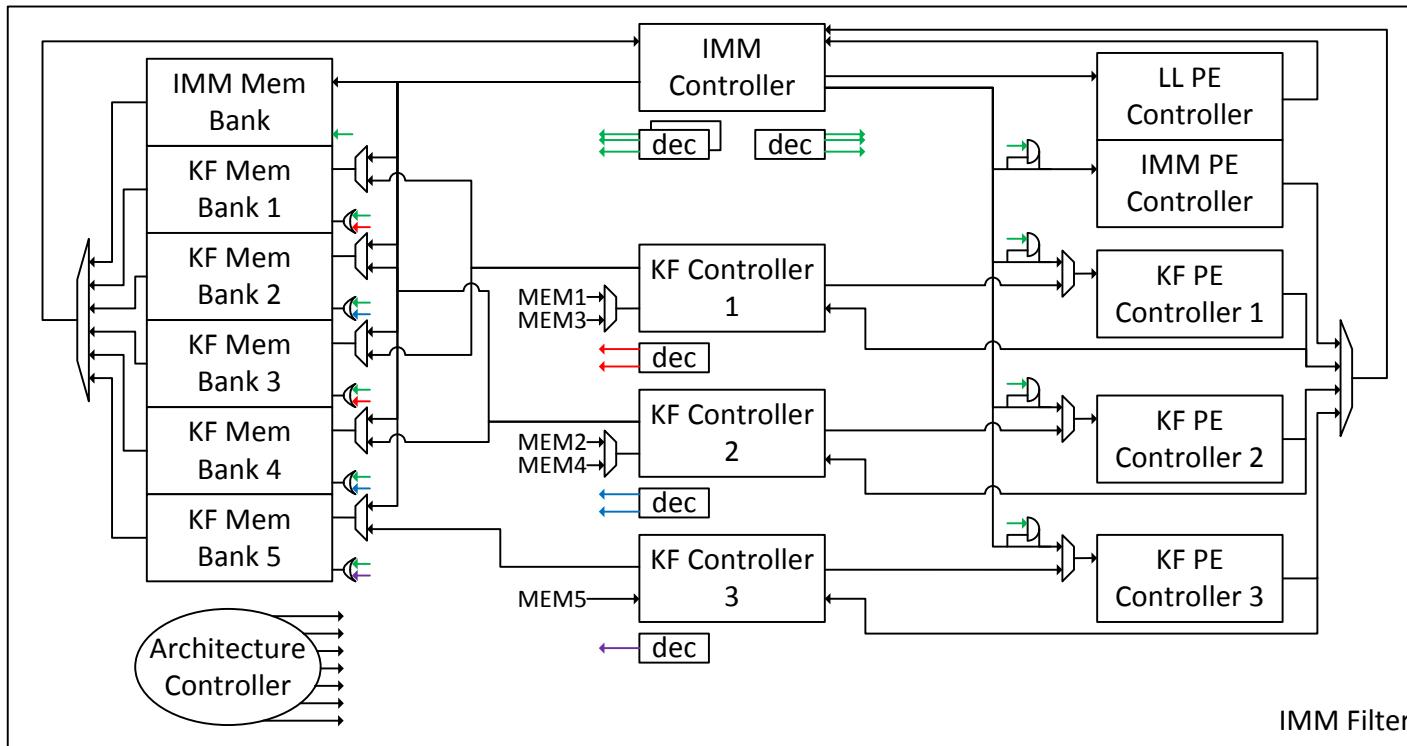
Clear Sky, Rain Fade,  
Ionospheric Scintillation,  
Tropospheric Scintillation,  
Multipath Fading



# IMM Analysis



# Summary



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